

1. EJERCICIOS

1. Calcular el dominio de las siguientes funciones reales de varias variables reales:

$$1.1 \quad f(x, y) = \frac{\sqrt{9-x^2}}{y-2x}$$

$$1.2 \quad f(x, y) = (\sqrt{x-y}, \ln(x-2), \frac{1}{x-y})$$

$$1.3 \quad f(x, y) = (\sqrt{x^2 + y^2 - 4}, \ln(16 - x^2 - y^2), \ln(x) + \ln(y))$$

2. Calcular

$$\lim_{(x,y) \rightarrow (0,0)} (x^2 + y^2, (x^2 + y) \operatorname{sen}(xy), x^2 + yx)$$

3. Calcular los siguientes límites en el caso de que existan:

$$3.1. \quad \lim_{(x,y) \rightarrow (1,2)} \frac{3-x+y}{4+x-2y}$$

$$3.2. \quad \lim_{(x,y) \rightarrow (4,\pi)} x^2 \operatorname{sen} \frac{y}{x}$$

$$3.3. \quad \lim_{(x,y) \rightarrow (0,0)} \frac{3x-2y}{2x-3y}$$

$$3.4. \quad \lim_{(x,y) \rightarrow (0,0)} \frac{2x-y}{x^2+y^2}$$

$$3.5. \quad \lim_{(x,y) \rightarrow (0,0)} \frac{x}{x+y}$$

$$3.6. \quad \lim_{(x,y) \rightarrow (0,0)} x^2 y^2 \ln(x^2 + y^2)$$

$$3.7. \quad \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^2 + y^2}$$

$$3.8. \quad \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{(x^2 + y^2)^{3/2}}$$

$$3.9. \quad \lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2 + y^2}$$

$$3.10. \quad \lim_{(x,y) \rightarrow (0,0)} \frac{x|y|}{\sqrt{x^2 + y^2}}$$

$$3.11. \quad \lim_{(x,y) \rightarrow (0,0)} \frac{(y^2 - x)^2}{x^2 + y^4}$$

$$3.12. \quad \lim_{(x,y) \rightarrow (0,0)} \frac{(y^2 - x)^2}{x^2 + y^2}$$

$$3.13. \quad \lim_{(x,y) \rightarrow (0,0)} \frac{y}{x} \operatorname{sen}\left(\frac{1}{x^2 + y^2}\right)$$

$$3.14. \quad \lim_{(x,y) \rightarrow (0,0)} \frac{\operatorname{sen}(x-y)}{\sqrt{x^2 + y^2}}$$

$$3.15. \quad \lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xy^2z}{x^2 + y^2 + z^2}$$

4. Representar gráficamente las superficies de nivel de las siguientes funciones:

$$4.1. \quad f(x, y, z) = 4x^2 + y^2 + 4z^2 \qquad 4.2. \quad f(x, y, z) = x^2 + y^2 - z^2$$

$$4.3. \quad f(x, y, z) = (x - a)^2 + (y - b)^2$$

5. Analizar la continuidad de las siguientes funciones:

$$5.1. \quad f(x, y) = \begin{cases} \frac{x^2 y^2}{x^2 y^2 + (x-y)^2} & \text{si } (x, y) \neq (0, 0) \\ 0 & \text{si } (x, y) = (0, 0) \end{cases}$$

$$5.2. \quad f(x, y) = \begin{cases} \frac{x^2 - y^2}{x^2 + y^2} & \text{si } (x, y) \neq (0, 0) \\ 0 & \text{si } (x, y) = (0, 0) \end{cases}$$

$$5.3. \quad f(x, y) = \begin{cases} (x + y)^2 \cos\left(\frac{1}{x^2 + y^2}\right) & \text{si } (x, y) \neq (0, 0) \\ 0 & \text{si } (x, y) = (0, 0) \end{cases}$$

$$5.4. \quad f(x, y) = \begin{cases} (x^2 + y^2) \cos\left(\frac{1}{x^2 + y^2}\right) & \text{si } (x, y) \neq (0, 0) \\ 0 & \text{si } (x, y) = (0, 0) \end{cases}$$

$$5.5. \quad f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2} & \text{si } (x, y) \neq (0, 0) \\ 0 & \text{si } (x, y) = (0, 0) \end{cases}$$

$$\begin{aligned} 5.6. f(x, y) &= \begin{cases} \frac{x}{x+y} & \text{si } x + y \neq 0 \\ 0 & \text{si } x + y = 0 \end{cases} \\ 5.7. f(x, y) &= \begin{cases} x^2 \operatorname{sen}\left(\frac{1}{y}\right) & \text{si } y \neq 0 \\ x^2 & \text{si } y = 0 \end{cases} \\ 5.8. f(x, y) &= \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & \text{si } (x, y) \neq (0, 0) \\ 0 & \text{si } (x, y) = (0, 0) \end{cases} \\ 5.9. f(x, y) &= \begin{cases} \left(\frac{x^2y}{x^2+y^2}, \operatorname{sen} xy\right) & \text{si } (x, y) \neq (0, 0) \\ (0, 0) & \text{si } (x, y) = (0, 0) \end{cases} \end{aligned}$$